GRAVITATIONAL FLOW OF GRANULAR MATERIALS

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The free flow of granular materials through an orifice in a horizontal bottom has been investigated. An equation is proposed for calculating the mass flow rate.

In industrial equipment granular materials are often required to flow under gravity in various kinds of vertical tubes, pipes, and spouts, as well as through openings in horizontal bottoms.

Since the initial investigation [1] many studies of the gravitational flow of granular materials have been published and dozens of quantitative laws for calculating the rate of flow of such materials through orifices have been proposed [2, 3].

These equations differ mathematically and with respect to the parameters they contain. The results they give frequently do not coincide. This presents certain difficulties in connection with the choice of a suitable quantitative relation for determining the mass flow rate of granular material, complicated by the conflicting opinions on the effect of the individual parameters expressed by different investigators and the limited applicability of most of the proposed equations. This has necessitated a further investigation of the gravitational flow of granular materials.

We already know that the flow rate is not affected by the thickness of the bed, if it is greater than (0.5-1) times the diameter of the orifice [2, 4-7], by the size of the vertical channel, if its ratio to the diameter of the orifice is greater than three [5, 8], or by the slope of the walls of a conical funnel, if it is less than 60° [5, 9, 10].

It is also known that at $D/d < 7$ the flow may become irregular or cease completely [2, 5, 8-10]. It should be noted that the flow of materials consisting of particles smaller than 0.15 mm has scarcely been investigated and is evidently subject to different laws because of the considerable ejecting capacity of such materials [5, 11].

We must now consider, in the light of the above, the general criterial equation proposed in [12] for the gravitational flow of granular materials, which for the case of flow into an air medium can be written in the simplified form:

$$\mathrm{Fr} = f(D/d), \tag{1}$$

obtained on the assumption that the angle of repose of the granular material does not affect the gravitational flow rate.

TABLE 1. Characteristics of Granular Materials

Material	d, mm	$\gamma_b \cdot 10^{-3}$, N/m^3	$\beta°$
Slag pellets	1,4	16,7	25
Millet	2,0	8,4	29
Iron shot	0,5	41,6	29
River sand	0,23	13,4	33
River sand	0,33	13,4	33
River sand	0,80	14,2	33
Corundum	0,30	18,8	37

We have investigated the gravitational flow of various granular materials (Table 1) through an orifice in a horizontal bottom. The diameter of the orifice varied from 2 to 60 mm.

Most of the experiments were performed using a vertical Plexiglas channel 1000 mm tall with a cross-sectional area of 100 × 100 mm. In investigating flow through orifices 40 and 60 mm in diameter we used a vertical channel with a cross section of 200 × 200 mm.

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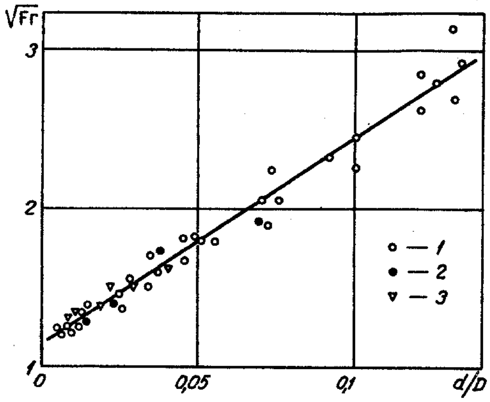

Fig. 1

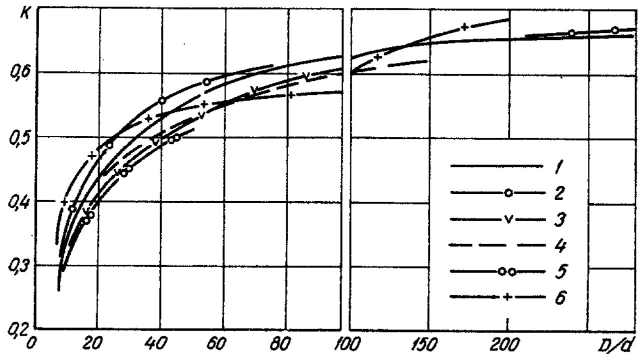

Fig. 2

Fig. 1. Generalization of the experimental data: 1) our data; 2) data from Table 2 of [15]; 3) data of [7].

Fig. 2. Graphs of the equations of various authors: 1) our equation (2); 2) Keneman's equation [5]; 3) Rausch's equation [2]; 4) Tanak's equation [9]; 5) equation of the authors of [9]; 6) Kuong's equation [20].

The method of investigation was the usual one and consisted in determining the amount of granular material passing in a measured interval of time through an orifice of given diameter. In calculating the rate of flow we used the bulk density of the granular material. The time interval was selected in the interval 1-5 min depending on the diameter of the orifice.

It was established that the rate of flow does not depend on the angle of repose. The same conclusion was reached by the authors of [13, 14], who worked with granular materials whose angle of repose varied in the ranges of 19.5-34° [13] and 19-37° [14], by Keneman [5], who investigated the flow of granular materials with an angle of repose from 18 to 36°, and also by the authors of [8]. In [15] an equation was derived according to which the rate of flow does not depend on the angle of repose and whose reliability was confirmed by comparison with the experimental data for materials with β = 23-41°. Admittedly, in that study it was suggested that the flow rate might be affected by the angle of repose if the range of variation of the latter was broadened. According to the opinion of the author of [16], based on a theoretically derived equation, it is possible to neglect the effect of the angle of repose on the flow rate of easily flowing materials at β = 22-50°.

Certain authors express contrary opinions. Thus, Rausch [2], having studied the flow of materials with an angle of repose between 23 and 41°, states that the mass flow rate is inversely proportional to $(\tan\beta)^{0.5}$. The authors of [9] present an equation of Tanak's according to which this quantity is inversely proportional to $(\tan\beta)^{0.32}$. The authors of [17], who investigated the flow of a material with β = 32-38°, assume that the gravitational flow rate depends on a mobility factor determined by the angle of repose.

An analysis of the equations of Rausch, Tanak, and the authors of [17] showed that calculating the flow velocity without allowance for the effect of the angle of repose, in accordance with the mean value of that angle on the investigated interval, introduces a maximum error of ±25, ±12, and ±10%, respectively. Obviously, this error is relatively large only in the case of Rausch's equation, while in the case of the other authors it is close to the error of the experiment itself.

Thus, an effect of the angle of repose of the mass flow rate has either not been detected or has been found to be not very considerable. We may therefore assume that this angle does not have an important influence on the rate of gravitational flow of a granular material through an orifice in a horizontal bottom.

Accordingly, using Eq. (1), we can generalize the experimental data obtained for the gravitational flow of various granular materials into an air medium under the above-mentioned conditions. As a result of this generalization (Fig. 1) we obtain the equation

$$\sqrt{\mathrm{Fr}} = 1.14 + 13.4\ d/D, \tag{2}$$

whose mean error is ±5%.

The graphs representing the equations of various authors are plotted in Fig. 2 in D/d, K coordinates, in which the experimental data have frequently been generalized in previous publications and which therefore facilitate the construction of the graphs. It is easy to see that $K = 4/\pi\sqrt{Fr}$. Each graph is plotted in the range of the ratio D/d investigated by the corresponding author. Where necessary, the angle of repose has been taken equal to the mean value for the materials investigated by the particular author.

It should be noted in passing that in the Rausch equation presented in [18, 19] the coefficient 0.19 has been omitted, i.e., the formula should read:

$$G = \frac{0.19\, CC_0}{\sqrt{\mathrm{tg}\,\beta}}\; g^{0.5}\gamma_b d^{2.5}\left(\frac{D}{d}\right)^{2.7}, \tag{3}$$

which is obvious from the graph of Rausch's data presented in [2]. Curve 3 and Fig. 2 corresponds to the Rausch equation given in this article.

Equation (2) can be transformed to the more convenient form:

$$G = \frac{2.15}{1 + 11.8\,(d/D)}\,\gamma_b D^{2.5}, \tag{4}$$

where D and d are in m, γ_b in N/m^2, and G in N/sec.

Equation (4) thus obtained, which is simple in structure and sufficiently accurate, as indicated by Figs. 1 and 2, can be used over the entire practical range of the ratio D/d (from 7 to 300) with the above-mentioned restrictions ($d > 0.15$ mm, $\alpha < 60°$, $D_c/D > 3$, thickness of bed of material greater than diameter of orifice).

In using Eqs. (2) and (4) it should be taken into account that they were obtained from an investigation of the flow of granular materials with rounded particles and that the materials themselves were so-called ideal granular media, i.e., materials with low or zero interparticle adhesion.

NOTATION

C, C_0	are coefficients;
D	is the diameter of outlet orifice;
D_c	is the diameter of vertical channel;
$Fr = gD/w^2$	is the Froude number;
G	is the mass rate of flow of granular material through orifice per unit time;
g	is the acceleration of gravity;
$K = G/\gamma_b D^{2.5} g^{0.5}$	is a dimensionless parameter;
w	is the rate of flow of granular material;
α	is the angle of inclination of funnel walls to horizontal;
β	is the angle of repose;
γ_b	is the bulk density of granular material.

LITERATURE CITED

1. E. Hagen, Berliner Monatsberichte der Akademie der Wissenschaften, 35, 1852.
2. In: Fluidized-Bed Cracking Catalysts [Russian translation], GOSINTI, Moscow (1958).
3. W. Zablotny and K. Akerman, Przem. Chem., 45, 152-159 (1966).
4. E. A. Banit and P. N. Platonov, Izvestiya VUZ Pishchevaya Tekhnologiya, No. 1 (1958).
5. F. E. Keneman, Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk Mekh. i Mashinostr., No. 2 (1960).
6. A. V. Anatol'ev, A. P. Kovtun, and P. N. Platonov, Izvestiya VUZ Pishchevaya Tekhnologiya, No. 4 (1961).
7. Ya. G. Mutovin and U. P. Dzyuba, Trudy GrozNII, 20, 241-250 (1966).
8. P. I. Luk'yanov, I. V. Gusev, and N. M. Nikitina, Khim. i Tekhnol. Topliv i Masel, No. 10 (1960).
9. J. Ciborowski and M. Badzynski, Chem. Stosow., 8, 289-315 (1963).
10. R. Kvapil, Motion of Granular Material in Hoppers [in Russian], Gosgortekhizdat (1961).
11. F. E. Keneman, N. G. Zalogin, V. N. Vorob'ev, and O. S. Antoshina, Inzhen.-Fiz. Zh., 3, No. 3 (1960).
12. Z. R. Gorbis, Heat Transfer of Disperse Throughflows [in Russian], Izd. Énergiya, Moscow – Leningrad (1964).

13. R. J. Fowler and J.R. Glastonbury, Chem. Eng. Sci., 10, 150-156 (1959).
14. J. E. Richardson and R. A. Weikle, Trans. Inst. Chem. Eng., 39, No. 5 (1961).
15. F. A. Zenz, Hydrocarbon Processing, Petroleum Refiner, 41, 159-168 (1962).
16. R. L. Zenkov, Mechanics of Loose Loads [in Russian], Izd. Mashinostroenie, Moscow (1964).
17. Yu. M. Borisov and L. Z. Khodak, Inzhen.-Fiz. Zh., 8, 712-719 (1965).
18. S. S. Zabrodskii, Hydrodynamics and Heat Transfer in a Fluidized Bed [in Russian], Gosénergoizdat, Moscow–Leningrad (1963).
19. M. Leva, Fluidization [Russian translation], Gostoptekhizdat, Moscow–Leningrad (1963).
20. I. F. Kuong, Brit. Chem. Eng., 8, 572 (1963).